A 3-Position Transmission/Reflection Method for Measuring the Permittivity of Low Loss Materials

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***Abstract*—A 3-position transmission/reflection method for measuring the permittivity of low loss materials is presented. In this method, the measurement errors of the permittivity due to 1) the mismatch at the connections, 2) the metallic loss of the holder, 3) the uncertainty of sample position in the airline, and 4) the imperfection of the calibration kit can be removed by the measured data for a sample at three different positions. Experimental results for a low loss engineering plastic ($\tan\delta \cong$ 0.025) are included.**

I. Introduction

The PERMITTIVITY of the low loss material is usually measured by the resonator methods. Using the method, the measurement frequency band is very narrow, and the accuracy is limited because the permittivity is obtained from perturbation formulas [1]. Hence, the transmission/reflection (TR) method is needed for broad-band measurement of material constants. Although the TR method is relatively simple, the measurement uncertainty of loss factor for low loss materials is high [2]. Thus, an improved TR method is required for accurately measuring the permittivity of low loss materials over a wide range of frequencies.

Enders [1] has shown that the measurement of three different lengths of an unknown line is sufficient to determine all the properties of the unknown line and those of the junctions to the measurement lines. In the TR method, the sample should be well prepared because the airgap between the sample and the outer or inner conductor gives rise to serious measurement errors [3]. Therefore, in using Enders' scheme for the TR method, it is hard to make three well-prepared samples of different lengths. Also, there is the possibility that the differences in the permittivity and permeability of each sample incur excessive measurement errors. Meanwhile, Baker-Jarvis [4] and we [5] have shown that the uncertainty of sample position in the airline causes serious measurement errors. Hence, a position insensitive TR method that requires only one sample is needed to obtain accurate results.

In this paper, we present an improved TR method with the use of known calibration techniques [6], [7]. The method can remove the errors due to the mismatch at the connections, the metallic loss of the holder, the uncertainty of sample position in the airline, and the imperfection of the calibration kit. The measurement algorithm for determining the permittivity using the measured data of a sample at three different positions is described in Section II. The experimental results are included in Section III.

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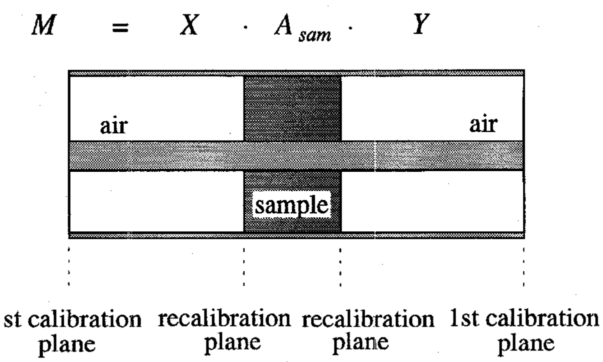

Fig. 1. Coaxial airline containing a dielectric sample and the calibration planes.

II. The 3-Position TR Method

A dielectric sample is placed in a section of coaxial airline as shown in Fig. 1. The total measurement system can be modelled by the product of an input error ABCD or transmission matrix X, the sample ABCD matrix A_{sam}, and an output error ABCD matrix Y [8]. The matrices X and Y contain the effects of the air part of the coaxial airline, the connections between the holder and automatic vector network analyzer (ANA) connector, and ANA IF mixer conversion factor, etc.

The measured ABCD matrix M of the sample can be written as

$$M = X \cdot A_{\text{sam}} \cdot Y. \tag{1}$$

Since it is assumed that the sample is homogeneous and linear and that the measurement frequency is limited such that the higher-order modes cannot propagate, the ABCD matrix for the sample of length d becomes

$$A_{\text{sam}} = \begin{bmatrix} \cosh\gamma d & Z\sinh\gamma d \\ Z^{-1}\sinh\gamma d & \cosh\gamma d \end{bmatrix}. \tag{2}$$

In (2), Z and γ are the characteristic impedance and propagation constant in the sample filled region, respectively. Z and γ can be represented by the permittivity ϵ and permeability μ of the sample as follows:

$$\gamma = j\omega\sqrt{\epsilon\mu} \tag{3}$$

$$Z = \frac{\log(b/a)\sqrt{\mu/\epsilon}}{2\pi}. \tag{4}$$

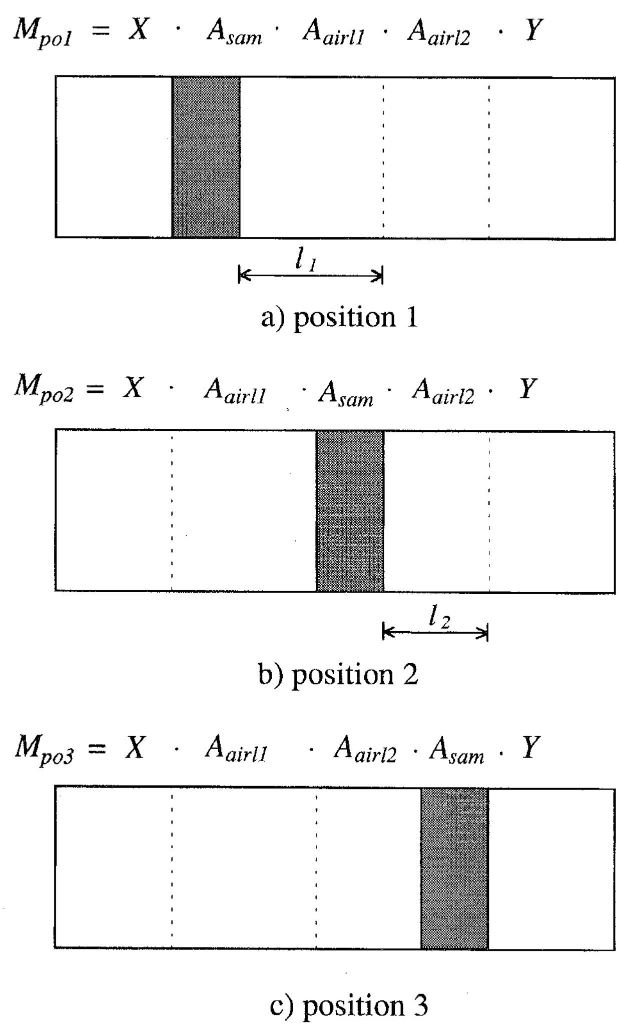

Fig. 2. Definitions of A_{airl1}, A_{airl2}, l_1, and l_2 when a sample is installed at (a) position 1, (b) position 2, and (c) position 3.

Here a and b are the outer radius of the inner conductor and the inner radius of the outer conductor of the airline, respectively, and ω is the measured angular frequency. Since the measured sample is dielectric, the permeability of the sample is equal to that of vacuum.

The 3-position TR method is based on the ABCD matrix transformed from the scattering matrix for three different sample positions. The transformed ABCD matrix M_{po1} is for position 1 and the other matrices M_{po2} and M_{po3} are for positions 2 and 3, respectively, as shown in Fig. 2.

Multiplying the matrix M_{po1} by the inverse matrix of M_{po2}, we obtain

$$M_{\mathrm{po1}} \cdot M_{\mathrm{po2}}^{-1} = X \cdot A_{\mathrm{sam}} \cdot A_{\mathrm{airl1}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl1}}^{-1} \cdot X^{-1}. \quad (5)$$

In (5), A_{airl1} is the ABCD matrix for the air region of length l_1, which is equal to the difference between position 1 and position 2 as shown in Fig. 2(a) and (b), and X is the input error matrix when the sample is located at position 1. In the same way, multiplying M_{po2} by the inverse of M_{po3} and multiplying M_{po1} by the inverse of M_{po3} we obtain

$$M_{\mathrm{po2}} \cdot M_{\mathrm{po3}}^{-1} = X \cdot A_{\mathrm{airl1}} \cdot A_{\mathrm{sam}} \cdot A_{\mathrm{airl2}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl2}}^{-1} \cdot (X \cdot A_{\mathrm{airl1}})^{-1} \quad (6)$$

$$M_{\mathrm{po1}} \cdot M_{\mathrm{po3}}^{-1} = X \cdot A_{\mathrm{sam}} \cdot A_{\mathrm{airl1}} \cdot A_{\mathrm{airl2}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl1}}^{-1} \cdot A_{\mathrm{airl2}}^{-1} \cdot X^{-1}. \quad (7)$$

In (6) and (7), A_{airl2} is the ABCD matrix for the air region of length l_2, that is equal to the difference between position 2 and position 3 as shown in Fig. 2(b) and (c).

We note from (5) that the matrix $M_{\mathrm{po1}} \cdot M_{\mathrm{po2}}^{-1}$ is similar to the matrix $A_{\mathrm{sam}} \cdot A_{\mathrm{airl1}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl1}}^{-1}$. Also, we note from (6) and (7) that $M_{\mathrm{po2}} \cdot M_{\mathrm{po3}}^{-1}$ and $M_{\mathrm{po1}} \cdot M_{\mathrm{po3}}^{-1}$ are similar to $A_{\mathrm{sam}} \cdot A_{\mathrm{airl2}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl2}}^{-1}$ and $A_{\mathrm{sam}} \cdot A_{\mathrm{airl1}} \cdot A_{\mathrm{airl2}} \cdot A_{\mathrm{sam}}^{-1} \cdot A_{\mathrm{airl1}}^{-1} \cdot A_{\mathrm{airl2}}^{-1}$. Using the fact that two similar matrices have the same trace, which is defined as the sum of the diagonal elements of a matrix [9], and with the help of (2) we obtain the traces of matrices in (5)–(7) as follows:

$$\sin^2(\beta d\sqrt{\epsilon_r})\sin^2(\beta l_1)\left(\sqrt{\epsilon_r} - \frac{1}{\sqrt{\epsilon_r}}\right)^2 + 2 = \mathrm{Trace}\left(M_{\mathrm{po1}} \cdot M_{\mathrm{po2}}^{-1}\right) \quad (8)$$

$$\sin^2(\beta d\sqrt{\epsilon_r})\sin^2(\beta l_2)\left(\sqrt{\epsilon_r} - \frac{1}{\sqrt{\epsilon_r}}\right)^2 + 2 = \mathrm{Trace}\left(M_{\mathrm{po2}} \cdot M_{\mathrm{po3}}^{-1}\right) \quad (9)$$

$$\sin^2(\beta d\sqrt{\epsilon_r})\sin^2[\beta(l_1 + l_2)]\left(\sqrt{\epsilon_r} - \frac{1}{\sqrt{\epsilon_r}}\right)^2 + 2 = \mathrm{Trace}\left(M_{\mathrm{po1}} \cdot M_{\mathrm{po3}}^{-1}\right). \quad (10)$$

In (8)–(10), β is the phase propagation constant for the air region and ϵ_r is the complex relative permittivity of the sample. To determine ϵ_r from (8), the position information l_1 should be found. Manipulating (8)–(10), we obtain

$$\sin^2(\beta l_1) = 1 - \left(\frac{A}{2} + \frac{1}{2A} - \frac{1}{2AB^2}\right)^2 \quad (11)$$

where the constants A and B are given as

$$A = \sqrt{\frac{\mathrm{Trace}\left(M_{\mathrm{po1}} \cdot M_{\mathrm{po3}}^{-1}\right) - 2}{\mathrm{Trace}\left(M_{\mathrm{po2}} \cdot M_{\mathrm{po3}}^{-1}\right) - 2}} \quad (12)$$

$$B = \sqrt{\frac{\mathrm{Trace}\left(M_{\mathrm{po2}} \cdot M_{\mathrm{po3}}^{-1}\right) - 2}{\mathrm{Trace}\left(M_{\mathrm{po1}} \cdot M_{\mathrm{po2}}^{-1}\right) - 2}}. \quad (13)$$

When (11) is substituted into (8), the complex relative permittivity ϵ_r of the sample can easily be determined.

The physical meaning of this procedure is that the calibration plane is shifted from the front of the airline to the front of the sample by recalibration (in Fig. 1). Therefore, the errors arising in that region and the errors due to the imperfection of the calibration kit can be removed. Note also the 3-position TR method is independent of the sample position in the airline.

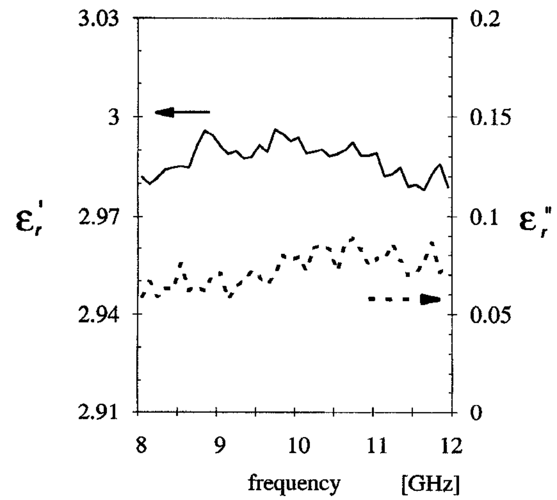

Fig. 3. Real (ϵ_r') and imaginary (ϵ_r'') parts of the permittivity of an engineering plastic sample obtained by the 3-position TR method in the X band.

III. Experiment Results and Discussions

The permittivity of a low loss Engineering Plastic sample with the length of 3.555 mm was measured. In so doing, the S-parameters of the sample were measured at four arbitrary positions in the airline. In this way, four possible results can be obtained. The final results shown in Fig. 3 are the average values. The relative permittivity of the Engineering Plastic is about 2.985-j0.07 in the X band, and the standard deviation of real and imaginary parts are 0.00491 and 0.00856, respectively. The deviations of the measured data result from the imperfection of the sample preparation and errors in measuring the magnitude and phase of the scattering parameters.

It should be noted that the 3-position TR method can also be applied to the measurement system in which a waveguide holder is used instead of the airline holder. Since this method can eliminate the effects of the coaxial to waveguide adapters, it has an advantage of not requiring the waveguide calibration kit for each frequency band.

IV. Conclusion

We have shown that the errors arising in permittivity measurement due to the mismatch at the connections, the metallic loss of coaxial airline, and the imperfection of the calibration kit can be removed by the measured data for a sample at three different positions. The method does not require the knowledge of the sample position in coaxial holder. The proposed method is particularly useful for characterizing low loss dielectric materials, for which the metallic loss of the holder and the mismatch at the connections become important.

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